Fig.1

Fig.2

Fig.3 (a)

$$P(\sigma_{xy})$$

Fig.3 (b)

Fig.3 (c)

Fig.3 (d)

$P(\sigma_{xx})$
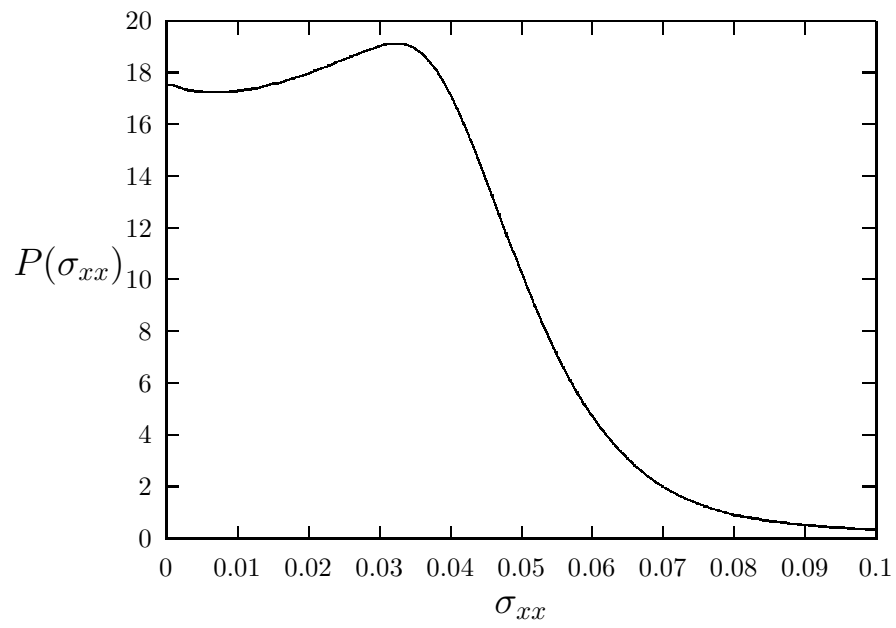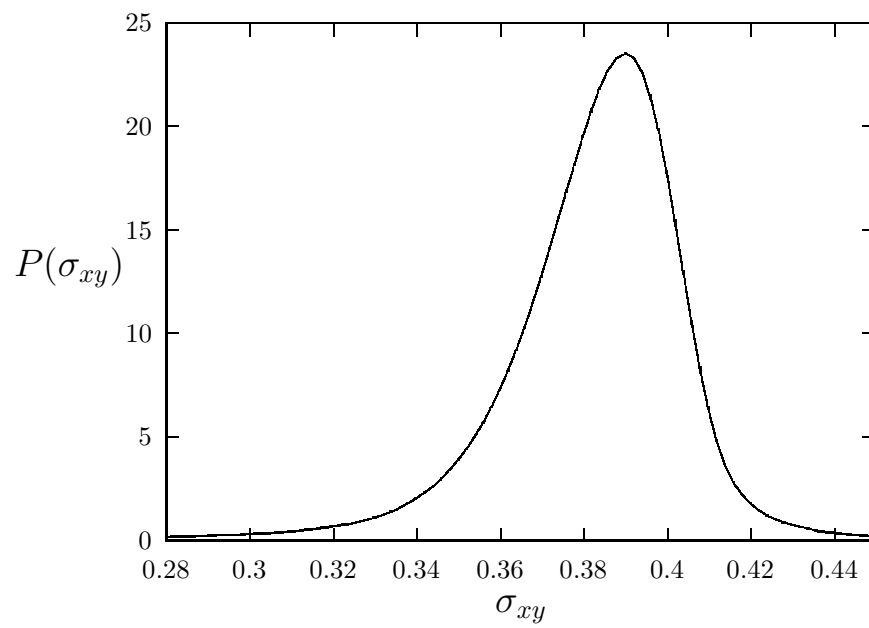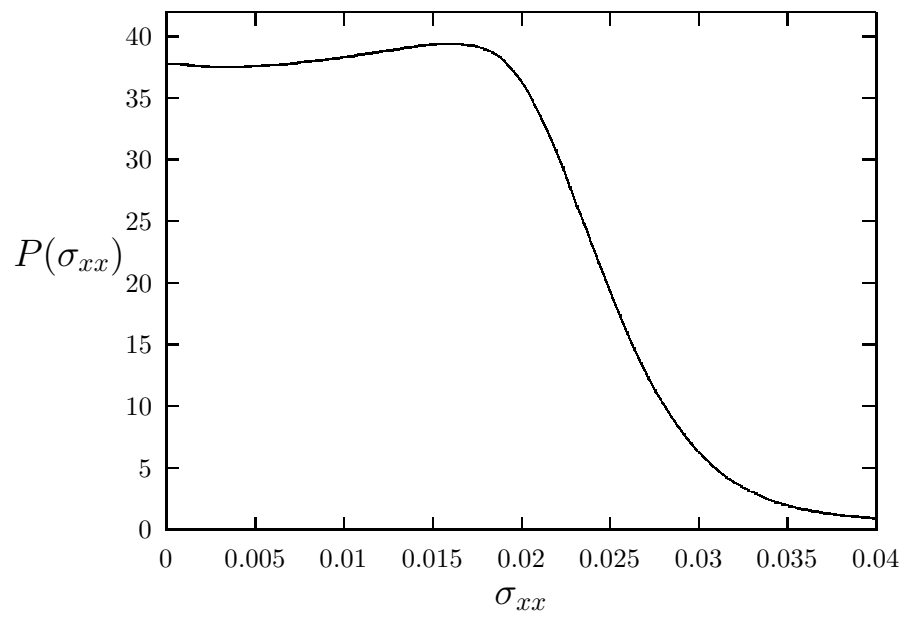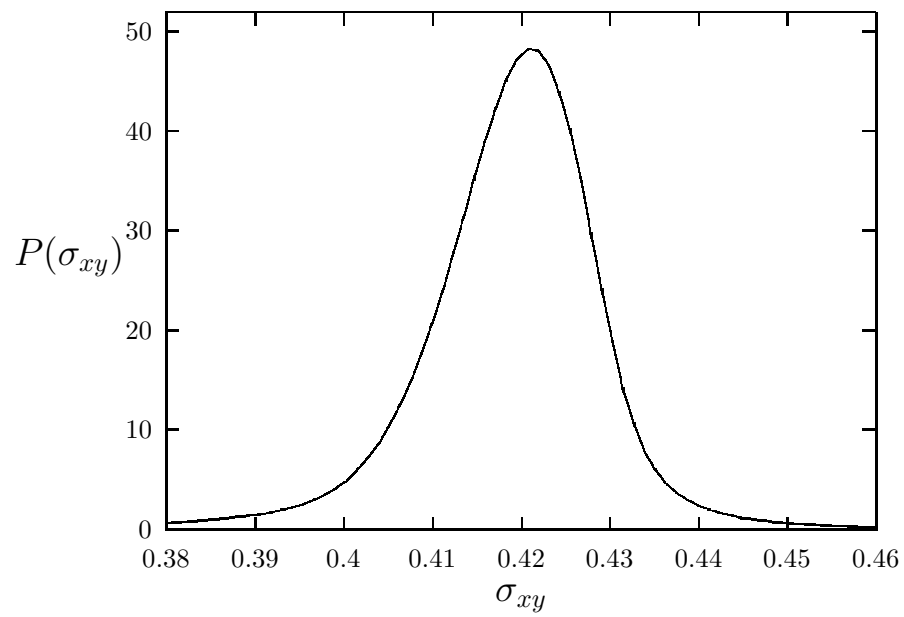
Fig.3 (e)

Fig.3 (f)

Conductance fluctuations
at the fractional quantum Hall plateau transitions

Hae-Young Kee[a,b], Yong Baek Kim[a,b], Elihu Abrahams[a], and R. N. Bhatt[b,c]

[a]Serin Physics Laboratory, Rutgers University, Piscataway, NJ 08855-0849
[b]Bell Laboratories, Lucent Technologies, Murray Hill, NJ 07974
[c]Department of Electrical Engineering, Princeton University, Princeton, NJ 08544

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We obtain a "mean field" scaling flow of the longitudinal and the Hall conductivities in the fractional quantum Hall regime. Using the composite fermion picture and assuming that the composite fermions follow the Khmelnitskii-Pruisken scaling flow for the integer quantum Hall effect, the unstable fixed points which govern the transitions between different fractional quantum Hall states are identified. Distributions of the critical longitudinal and Hall conductivities at the unstable fixed points are obtained and implications of the results for the experiments on mesoscopic quantum Hall systems are discussed.

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Many interesting phenomena in disordered mesoscopic metals occur when the phase coherence length of the electrons exceeds the sample size [1]. In particular, these mesoscopic metals show sample specific conductance fluctuations in contrast to macroscopic systems where self-averaging leads to a disorder averaged conductance [1,2]. It turns out that the magnitude of the fluctuation is of the order of e^2/h and universal in the sense that it only depends on the symmetry of the problem. Thus, it has acquired the name "universal conductance fluctuations" [2].

On the other hand, in two-dimensional macroscopic electronic systems in high perpendicular magnetic fields (quantum Hall effect regime), metallic behavior can be observed only near quantum phase transitions, *i.e.*, the transitions between different quantum Hall plateaus [3]. At these quantum critical points, the disorder averaged longitudinal (σ_{xx}) and Hall conductivities (σ_{xy}) are expected to be universal [4,5]. These universal critical conductivities can be observed when the sample size becomes larger than the phase coherence length.

For the integer quantum Hall effect, Khmelnitskii and then Pruisken [6] suggested a two parameter scaling flow in terms of σ_{xx} and σ_{xy}. It was noticed that a non-linear sigma model with a topological term can describe the quantum phase transitions between different plateaus and the topological term is responsible for the metallic behavior at the quantum critical points [6]. According to this scaling flow, there are two types of fixed points. There are stable fixed points which correspond to the integer quantum Hall states with $(\sigma_{xx}, \sigma_{xy}) = (0, n)$ (in this paper, all the conductivities are written in units of e^2/h). There are also unstable fixed points which govern the critical behavior at the transitions between adjacent integer quantum Hall states. The disorder-averaged critical conductivities ($\langle \sigma_{xx} \rangle, \langle \sigma_{xy} \rangle$) at the $(0, n-1) \to (0, n)$ transition or at the corresponding unstable fixed points were suggested to be $(1/2, n-1/2)$. Here n is a positive integer. There exist numerical calculations [7,8] of $\langle \sigma_{xx} \rangle$ and $\langle \sigma_{xy} \rangle$ at the transition $(\sigma_{xx}, \sigma_{xy}) = (0, 0) \to (0, 1)$, which obtain $\langle \sigma_{xx} \rangle \approx 0.5$ and $\langle \sigma_{xy} \rangle \approx 0.5$.

Besides the average value, it is of interest to determine the conductance fluctuations near the critical point, which can be observed in mesoscopic quantum Hall samples where the phase coherence length becomes larger than the sample dimensions. More generally, one is interested in the probability distribution of the conductivities at these quantum critical points, which may be expected to be universal.

A study of the Hall conductivity at the transition between the state with $(\sigma_{xx}, \sigma_{xy}) = (0, 0)$ and the integer quantum Hall state with $(\sigma_{xx}, \sigma_{xy}) = (0, 1)$ was carried out by Huo and Bhatt [9]. They found that the Hall conductivity distribution is universal, independent of sample size, and symmetric about $\sigma_{xy} \approx 0.5$, but has long power-law tails.

Recently, experiments measuring two-terminal conductance provide further motivation [10]. Cobden and Kogan [10] measured the two-terminal conductance (which corresponds to σ_{xx} [11]) of mesoscopic samples in the integer quantum Hall regime. They found large conductance fluctuations near the integer quantum Hall plateau transitions indicating a broad distribution of the longitudinal conductance. In particular, they found that the distribution is almost uniform in the interval between zero and one in units of e^2/h.

There have been theoretical efforts to understand these large fluctuations of σ_{xx} [11]. Wang, Jovanovic, and Lee [12] have calculated the ensemble averaged two-terminal conductance and its fluctuations at the critical point of the integer quantum Hall plateau transitions. They used the Chalker-Coddington network model [13] and periodic boundary conditions in the transverse direction. They concluded that the average and all the higher moments of the conductance distribution are universal at the transition. It means that the entire distribution is universal. At the

same time, the distribution turns out to be very broad in the sense that there is no well-defined typical value. Cho and Fisher [14] also calculated the distribution of the conductance in terms of the network model with periodic and open boundary conditions. It was explicitly shown that the conductance is more or less uniformly distributed between zero and one and there is almost no weight for the conductances larger than one. Thus, both of the results [12,14] are consistent with the experiment of Cobden and Kogan [10].

These results naturally leads us to ask what the distributions of the longitudinal and the Hall conductivities are at the critical points for the *fractional* quantum Hall plateau transitions. At best, the theoretical calculations mentioned above can be applied only to the integer quantum Hall effect because the electron-electron interaction is not included. Since the electron-electron interaction is essential for the fractional quantum Hall effect, the calculation of the distributions of the conductivities at the critical points in the fractional quantum Hall regime requires the consideration of both the electron-electron interaction and disorder and is thus much more complicated.

Several years ago, Jain [15] proposed the composite fermion theory of the fractional quantum Hall effect. A composite fermion is obtained by attaching an even number $2m$ of fictitious flux quanta to an electron. At the mean field level, one takes into account only the average of the fictitious magnetic field due to the attached fictitious magnetic flux. Then the system can be described as fermions in an effective magnetic field $\Delta B = B - \tilde{B}$, where $\tilde{B} = 2mn_ehc/e$ is the averaged fictitious magnetic field and n_e is the density of electrons. Therefore, in the mean field approximation, the fractional quantum Hall states with $\nu = p/(2mp+1)$ can be described as integer quantum Hall states of composite fermions with p filled Landau levels occupied in an effective magnetic field ΔB [15,16]. Here ν is the filling fraction and p is an integer. The most important correlation effects due to the electron-electron interaction are supposed to be included in the construction of the composite fermions through the fictitious flux quanta.

In this paper, we use the composite fermion theory to get the scaling flow of the longitudinal and Hall conductivities, and the distributions of the conductivities at the critical points in the fractional quantum Hall regime. The main difference between the integer quantum Hall effect of the electrons and that of the composite fermions is that the composite fermions experience both potential disorder and random flux disorder while the electrons have only potential disorder [16]. The random flux disorder for the composite fermions arises due to the fact that the attached flux moves together with the electron so that an inhomogeous electron density distribution, which would occur in a random potential, induces a random fictitious magnetic field. As a first step, in the absence of a rigorous study about the effects of both types of disorder, we assume that the integer quantum Hall effect of composite fermions follows the Khmelnitskii-Pruisken scaling flow [6]. Using a relation between the conductivity tensor of the electrons and that of the composite fermions, we shall obtain the scaling flow in the fractional quantum Hall regime. In this scaling flow, the stable and the unstable fixed points are identified. Due to the mean-field nature of the connection between the integer and fractional quantum Hall states mentioned above, we call it a "mean field" scaling flow of the fractional quantum Hall effect. Assuming that the distributions of the critical conductivities of the composite fermions in their integer regime follow those of the electrons in their integer regime, we also get the distributions of the conductivities for the electrons at the critical points of the fractional quantum Hall plateau transitions.

The relation between the resistivity tensor ρ of the electrons and that $\rho_{\rm cf}$ of the composite fermions is given by [16]

$$\rho = \rho^{\rm cf} + \rho^{\rm cs} , \tag{1}$$

where

$$\rho^{\rm cs} = \begin{pmatrix} 0 & 2m \\ -2m & 0 \end{pmatrix} \tag{2}$$

comes from the Chern-Simons transformation. Then the longitudinal σ_{xx} and the Hall σ_{xy} conductivities of the electrons can be expressed in terms of the composite fermion conductivities $\sigma_{xx}^{\rm cf}$ and $\sigma_{xy}^{\rm cf}$ as follows.

$$\sigma_{xx} = \frac{\sigma_{xx}^{\rm cf} \, [\, (\sigma_{xx}^{\rm cf})^2 + (\sigma_{xy}^{\rm cf})^2 \,]}{(\sigma_{xx}^{\rm cf})^2 + [\, \sigma_{xy}^{\rm cf} + 2m((\sigma_{xx}^{\rm cf})^2 + (\sigma_{xy}^{\rm cf})^2) \,]^2}$$
$$\sigma_{xy} = \frac{[\sigma_{xy}^{\rm cf} + 2m((\sigma_{xx}^{\rm cf})^2 + (\sigma_{xy}^{\rm cf})^2) \,] \, [\, (\sigma_{xx}^{\rm cf})^2 + (\sigma_{xy}^{\rm cf})^2 \,]}{(\sigma_{xx}^{\rm cf})^2 + [\, \sigma_{xy}^{\rm cf} + 2m((\sigma_{xx}^{\rm cf})^2 + (\sigma_{xy}^{\rm cf})^2) \,]^2} \, . \tag{3}$$

These equations are valid for any realization of the disorder. For macroscopic samples, all the conductivities can be replaced by the disorder averaged value if the self-averaging is legitimate.

We assume that the integer quantum Hall effect of the composite fermions follows the Khmelnitskii-Pruisken scaling flow in Fig.1 [6]. There are stable fixed points $(\sigma_{xx}^{\rm cf}, \sigma_{xy}^{\rm cf}) = (0, n-1)$ which represent the integer quantum Hall states

of the composite fermions (here n is a positive integer). There are also unstable fixed points $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (1/2, n-1/2)$ which control the transitions bewteen the quantum Hall states $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (0, n-1)$ and $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (0, n)$.

In order to get the scaling flow for the fractional quantum Hall effect of the electrons, we use Eq.3 to map the Khmelnitskii-Pruisken flow in the $\sigma_{xx}^{\mathrm{cf}} - \sigma_{xy}^{\mathrm{cf}}$ plane to the flow in the $\sigma_{xx} - \sigma_{xy}$ plane. We consider only the principal sequence $\nu = p/(2p+1)$ of the fractional quantum Hall states, so $m = 1$ is taken from now on. Let us first calculate various limits. For any $\sigma_{xy}^{\mathrm{cf}}$, if $\sigma_{xx}^{\mathrm{cf}} \to \infty$, then $\sigma_{xx} \to 0$ and $\sigma_{xy} \to 1/2$. Thus, the entire line $\sigma_{xx}^{\mathrm{cf}} \to \infty$ goes to the point $(\sigma_{xx}, \sigma_{xy}) = (0, 1/2)$. The line $\sigma_{xx}^{\mathrm{cf}} = 0$ goes to $(\sigma_{xx}, \sigma_{xy}) = (0, \sigma_{xy}^{\mathrm{cf}}/(1 + 2\sigma_{xy}^{\mathrm{cf}}))$. Thus all the stable fixed points $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (0, n-1)$ go to

$$(\sigma_{xx}, \sigma_{xy}) = \left(0, \frac{n-1}{2(n-1)+1} \right) \tag{4}$$

which become also the stable fixed points in the $\sigma_{xx} - \sigma_{xy}$ plane and correspond to the fractional quantum Hall states of the electrons. On the other hand, all the unstable fixed points $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (1/2, n-1/2)$ can be mapped to

$$(\sigma_{xx}, \sigma_{xy}) = \left(\frac{1}{2} \frac{4n^2 - 4n + 2}{1 + (4n^2 - 2n + 1)^2}, \frac{1}{2} \frac{(4n^2 - 4n + 2)(4n^2 - 2n + 1)}{1 + (4n^2 - 2n + 1)^2} \right) . \tag{5}$$

For example, $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (1/2, 1/2), (1/2, 3/2)$, and $(1/2, 5/2)$ are mapped to $(\sigma_{xx}, \sigma_{xy}) = (1/10, 3/10), (1/34, 13/34)$, and $(13/962, 403/962)$. These are the critical conductivities at the transitions $(\sigma_{xx}, \sigma_{xy}) = (0,0) \to (0, 1/3), (0, 1/3) \to (0, 2/5)$, and $(0, 2/5) \to (0, 3/7)$ respectively. These critical conductivities have been also calculated in previous studies and are supposed to be valid for macroscopic samples [4,5]. As a last exercise, it can be easily seen that $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (0, n-1/2)$ are mapped to

$$(\sigma_{xx}, \sigma_{xy}) = \left(0, \frac{2n-1}{4n} \right) . \tag{6}$$

The resulting scaling flow for the principal sequence $\nu = p/(2p+1)$ is shown in Fig.2. It looks quite similar to the case of the integer quantum Hall states and is consistent with the selection rules in the law of corresponding states proposed by Kivelson, Lee, and Zhang [4]. For example, the direct transition between $(\sigma_{xx}, \sigma_{xy}) = (0,0)$ and $(0, 2/5)$ states is not allowed. The scaling flow also gives us some new information. For example, the scaling curve which starts at $(\sigma_{xx}, \sigma_{xy}) = (0, 1/2)$ and goes to $(1/10, 3/10)$ has a maximum at $(\sigma_{xx}, \sigma_{xy}) = (1/8, 3/8)$. This implies that the bare longitudinal conductivity of the smaple should be smaller than $1/8$ in order to see the $\nu = 1/3$ quantum Hall state. It is also interesting to notice that our result for the scaling flow in the fractional quantum Hall regime is different from that proposed by Laughlin *et al.* some years ago [17].

Now we consider the statistical properties of the conductivities at the critical points which are governed by the unstable fixed points given by Eq.5. Let us assume that the distributions of the critical conductivities for the integer quantum Hall effect of the composite fermions is the same as those for the electrons. Then the distribution of the critical longitudinal conductivities for the composite fermions at $(\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}) = (1/2, n-1/2)$ will be taken as [10–12,14]

$$P(\sigma_{xx}^{\mathrm{cf}}) = \begin{cases} 1 & 0 \leq \sigma_{xx}^{\mathrm{cf}} \leq 1 \\ 0 & \sigma_{xx}^{\mathrm{cf}} > 1 \ . \end{cases} \tag{7}$$

On the other hand, the distribution of the Hall conductivity for the composite fermions is taken as

$$P(\sigma_{xy}^{\mathrm{cf}}) = \begin{cases} 1.246 e^{-5.309(\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle)^2} \left[1 - 0.2791(\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle)^2 \right] & |\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle| \leq 0.5084 \\ 0.04122/(\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle)^{2.9} & |\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle| > 0.5084 \ , \end{cases} \tag{8}$$

where $\langle \sigma_{xy}^{\mathrm{cf}} \rangle = n - 1/2$. This distribution turns out to be an excellent parametrization of the numerical result [9]. Furthermore, both the distribution and its derivative at $|\sigma_{xy}^{\mathrm{cf}} - \langle \sigma_{xy}^{\mathrm{cf}} \rangle| = 0.5084$ are continuous. Notice that the same distribution is taken for any n with $\langle \sigma_{xy}^{\mathrm{cf}} \rangle = n - 1/2$ due to the invariance of the effective action or the non-linear sigma model [6] under $\sigma_{xy}^{\mathrm{cf}} \to \sigma_{xy}^{\mathrm{cf}} + l$, where l is an integer. One can also see that $P(\sigma_{xy}^{\mathrm{cf}})$ does not have a second moment due to the long tail.

Notice that the distributions of $\sigma_{xx}^{\mathrm{cf}}$ and $\sigma_{xy}^{\mathrm{cf}}$ may be correlated. In the absence of a rigorous study about the relation between these two distributions, in this paper, we assume that they are independent each other for simplicity. Then the distribution of σ_{xx} and σ_{xy} at the critical points of the fractional quantum Hall plateau transitions can be

3

obtained from the convolution of $P(\sigma_{xx}^{\mathrm{cf}})$ and $P(\sigma_{xy}^{\mathrm{cf}})$ using Eq.3. That is, the distribution $P(\sigma_{xx})$ and $P(\sigma_{xy})$ of σ_{xx} and σ_{xy} are given by

$$P(\sigma_{xx}) = \int_{-\infty}^{\infty} d\sigma_{xy} |J(\sigma_{xx}, \sigma_{xy} : \sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}})| P(\sigma_{xx}^{\mathrm{cf}}) P(\sigma_{xy}^{\mathrm{cf}})$$

$$P(\sigma_{xy}) = \int_{-\infty}^{\infty} d\sigma_{xx} |J(\sigma_{xx}, \sigma_{xy} : \sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}})| P(\sigma_{xx}^{\mathrm{cf}}) P(\sigma_{xy}^{\mathrm{cf}}) , \tag{9}$$

where $\sigma_{xx}^{\mathrm{cf}}$ and $\sigma_{xy}^{\mathrm{cf}}$ in the integrand should be written in terms of σ_{xx} and σ_{xy} from Eq.3. Here $J(\sigma_{xx}, \sigma_{xy} : \sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}})$ is the Jacobian for the change of the variables from $\sigma_{xx}^{\mathrm{cf}}$ and $\sigma_{xy}^{\mathrm{cf}}$ to σ_{xx} and σ_{xy}.

The results $P(\sigma_{xx})$ and $P(\sigma_{xy})$ at the critical point (or the unstable fixed point) for the transition $(\sigma_{xx}, \sigma_{xy}) = (0, 0) \to (0, 1/3)$ are shown in Fig.3 (a) and (b). One can see that $P(\sigma_{xx})$ has a maximum at $\sigma_{xx}^{\mathrm{max}} \approx 0.09$, but the distribution is still broad so that this is not really a typical value. Notice that there is almost no weight beyond $\sigma_{xx} \approx 0.5$. The average and the second moment are given by $\langle \sigma_{xx} \rangle = 0.1$ and $\delta \sigma_{xx} = \sqrt{\langle \sigma_{xx}^2 \rangle - \langle \sigma_{xx} \rangle^2} = 0.0725$. Thus the relative fluctuation is $\delta \sigma_{xx} / \langle \sigma_{xx} \rangle = 0.725$. On the other hand, $P(\sigma_{xy})$ has a well-defined typical value $\sigma_{xy}^{\mathrm{typical}} \approx 0.37$ which is different from the average value $\langle \sigma_{xy} \rangle = 0.3$. The reason why $P(\sigma_{xy})$ is skewed is that the relation Eq.3 between σ_{xx}, σ_{xy} and $\sigma_{xx}^{\mathrm{cf}}, \sigma_{xy}^{\mathrm{cf}}$ is non-linear. The cusp-like feature near $\sigma_{xy}^{\mathrm{typical}}$ is not a cusp and we can show that it is perfectly smooth if one plots $P(\sigma_{xy})$ for a limited interval, $e.g.$, between 0.35 and 0.4. The second moment and the relative fluctuation for σ_{xy} are given by $\delta \sigma_{xy} = 0.0675$ and $\delta \sigma_{xy} / \langle \sigma_{xy} \rangle = 0.225$ respectively.

For the transition $(\sigma_{xx}, \sigma_{xy}) = (0, 1/3) \to (0, 2/5)$, the results are shown in Fig.3 (c) and (d). $P(\sigma_{xx})$ is quite broad up to 0.1 with a maximum at $\sigma_{xx}^{\mathrm{max}} \approx 0.033$, but $P(\sigma_{xy})$ has a well-defined typical value $\sigma_{xy}^{\mathrm{typical}} \approx 0.39$ (note that $\langle \sigma_{xy} \rangle = 0.382$). The average and the second moment for σ_{xx} are given by $\langle \sigma_{xx} \rangle = 0.0294$ and $\delta \sigma_{xx} = 0.0184$. Thus the relative fluctuation is $\delta \sigma_{xx} / \langle \sigma_{xx} \rangle = 0.640$. On the other hand, the second moment and the relative fluctuation for σ_{xy} are given by $\delta \sigma_{xy} = 0.0220$ and $\delta \sigma_{xy} / \langle \sigma_{xy} \rangle = 0.0581$.

As the last example, we consider the transition $(\sigma_{xx}, \sigma_{xy}) = (0, 2/5) \to (0, 3/7)$, the results are shown in Fig.3 (e) and (f). $P(\sigma_{xx})$ is again very broad up to 0.04 with a maximum at $\sigma_{xx}^{\mathrm{max}} \approx 0.0164$, but $P(\sigma_{xy})$ has a well-defined typical value $\sigma_{xy}^{\mathrm{typical}} \approx 0.42$ while $\langle \sigma_{xy} \rangle = 0.419$. The average and the second moment for σ_{xx} are given by $\langle \sigma_{xx} \rangle = 0.0135$ and $\delta \sigma_{xx} = 0.00817$. Thus the relative fluctuation is $\delta \sigma_{xx} / \langle \sigma_{xx} \rangle = 0.613$. For σ_{xy}, the second moment and the relative fluctuation are given by $\delta \sigma_{xy} = 0.0111$ and $\delta \sigma_{xy} / \langle \sigma_{xy} \rangle = 0.0266$. All these results are summarized in Table 1.

Notice that both the relative fluctuations of σ_{xx} and of σ_{xy} are decreasing as n increases at the critical points given by Eq.5. However, the change of magnitude in the case of σ_{xy} is quite large while that in the case of σ_{xx} is very small. This implies that the distributions of σ_{xx} are almost equally broad for any n, but those of σ_{xy} become sharper and sharper as n is increased. Therefore, in experiments, it is expected that there will be large conductance fluctuations in σ_{xx} at any fractional quantum Hall plateau transition while the conductance fluctuations in σ_{xy} become smaller and smaller as the filling fraction of the quantum Hall states involved in the transition approaches $\frac{1}{2m} \frac{e^2}{h}$, where m is a positive integer.

We also calculated $P(\sigma_{xx})$ and $P(\sigma_{xy})$ using a Gaussian distribution for $\sigma_{xy}^{\mathrm{cf}}$. The purpose was to examine the effects of the long tail in the distribution obtained from the numerical calculation which we used above (see Eq.8). Comparing the results in the cases of using the Gaussian and the numerically-calculated distribution for $\sigma_{xy}^{\mathrm{cf}}$, we found that, in the latter case, the tails of $P(\sigma_{xx})$ and $P(\sigma_{xy})$ become indeed longer than those in the Gaussian case. However, the contributions from the tails are not large enough to give a diverging second moment so that the second moment of $P(\sigma_{xy})$ exists at the fractional quantum Hall plateau transitions in contrast to the case of the distribution of the Hall conductivity at the integer quantum Hall plateau transitions [9]. Other than these slightly longer tails, there is no qualitative difference between the two cases in the resulting $P(\sigma_{xx})$ and $P(\sigma_{xy})$.

In summary, a "mean field" scaling flow for the longitudinal and the Hall conductivities is obtained from the composite fermion theory assuming that the integer quantum Hall effect of the composite fermions follows the Khmelnitskii-Pruisken scaling flow. At the unstable fixed points which govern the transitions between the different fractional quantum Hall states, the distributions of the critical conductivities are obtained. It is found that the distributions of the longitudinal conductivity still remain as being broad and having almost no weight beyond certain value of the conductivity. The distributions of the Hall conductivity have well-defined typical values which are generally different from the average values and become sharper and sharper as the average Hall conductivity at the critical points approaches $\frac{1}{2m} \frac{e^2}{h}$.

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TABLE I. Summary of the results for the distributions of σ_{xx} and σ_{xy} at the fractional quantum Hall plateau transitions

	$\langle\sigma_{xx}\rangle$	$\langle\sigma_{xy}\rangle$	$\delta\sigma_{xx}$	$\delta\sigma_{xy}$	$\delta\sigma_{xx}/\langle\sigma_{xx}\rangle$	$\delta\sigma_{xy}/\langle\sigma_{xy}\rangle$
$0 \to 1/3$	0.1	0.3	0.0725	0.0675	0.725	0.225
$1/3 \to 2/5$	0.0294	0.382	0.0184	0.0220	0.640	0.0581
$2/5 \to 3/7$	0.0135	0.419	0.00817	0.0111	0.613	0.0266

FIG. 1. The Khmelnitskii-Pruisken scaling flow for the integer quantum Hall effect. In this paper, it is assumed that it can be applied to the integer quantum Hall effect of the composite fermions. □ and ○ correspond to the unstable and the stable fixed points.

FIG. 2. A "mean field" scaling flow for the fractional quantum Hall effect of the electrons. □ and ○ correspond to the unstable and the stable fixed points. We show only the part which has $\sigma_{xy} \leq 1/2$.

FIG. 3. (a), (c), and (e) are $P(\sigma_{xx})$s while (b), (d), and (f) are $P(\sigma_{xy})$s for the transitions $(\sigma_{xx}, \sigma_{xy}) = (0, 0) \to (0, 1/3)$, $(0, 1/3) \to (0, 2/5)$, and $(0, 2/5) \to (0, 3/7)$ respectively.